Physical Distancing to Prevent Influenza-Like-Illness on College Campuses: the eX-FLU Trial

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eX-FLU Trial

- **eX-FLU**: trial to evaluate a physical distancing intervention on a college campus during flu season (Aiello et al., 2016; Zivich et al., 2020)

Design and methods of a social network isolation study for reducing respiratory infection transmission: The eX-FLU cluster randomized trial

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- **Central question**: does the encouragement-to-isolate intervention prevent ILI infection?

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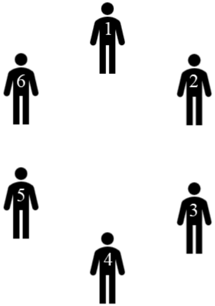
Transmission of Influenza-Like-Illness



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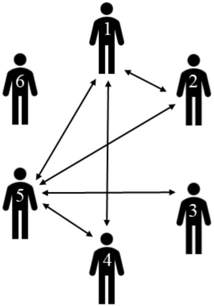

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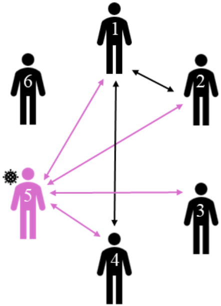

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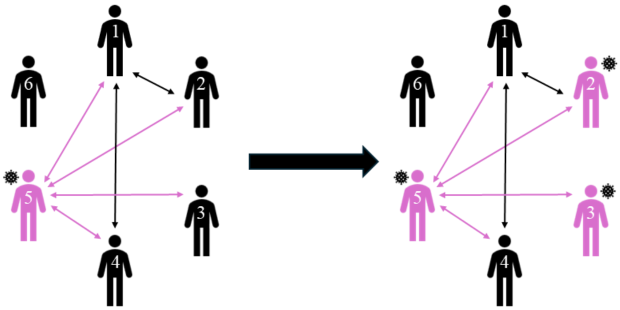
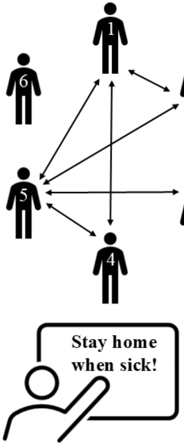


Example I: Intervention Affects Network and Infections



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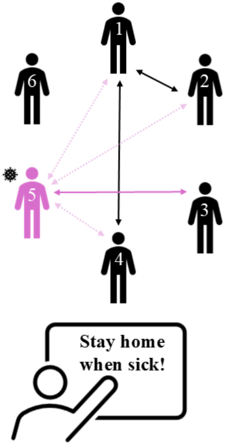

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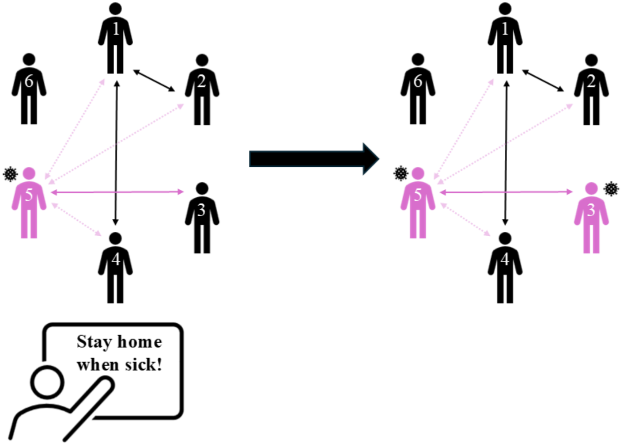

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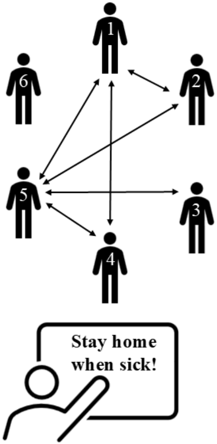

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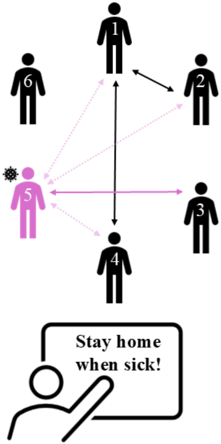

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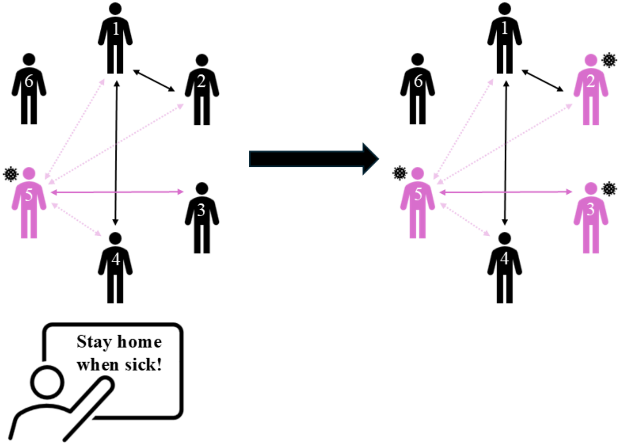

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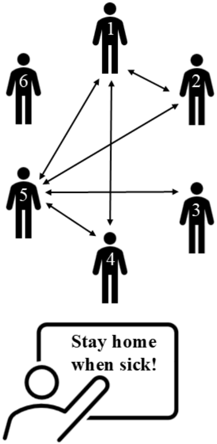

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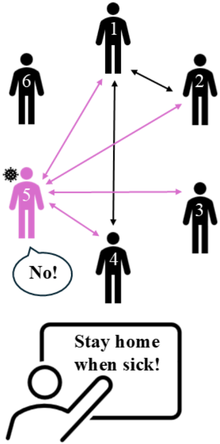

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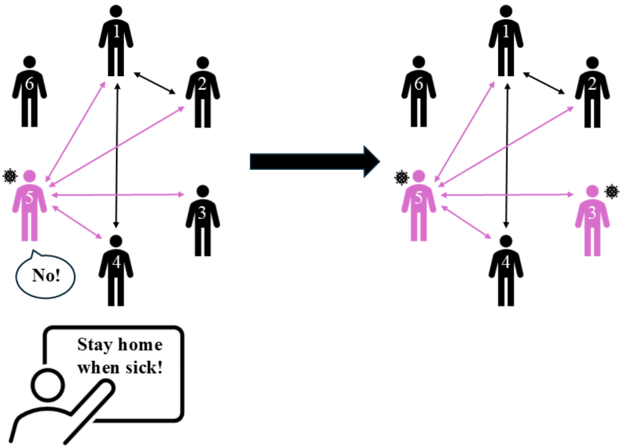
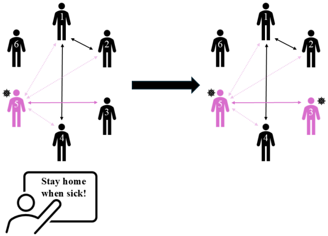


Central Question: Does the Intervention Prevent ILI Infection?

Example I:



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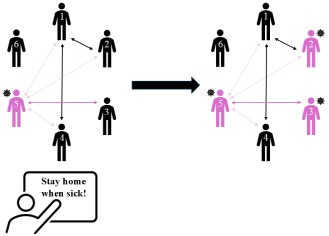

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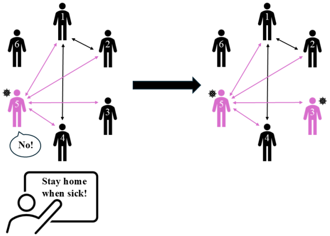
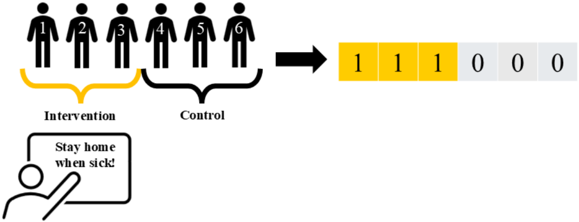


In Examples I and III, the answer is yes

eX-FLU Observed Data

- Baseline randomization assignments
  $\mathbf{Z} = (Z_1, \ldots, Z_n) \in \mathcal{Z}$, for
  $Z_i = \mathbb{1}(\text{student } i \text{ gets intervention})$



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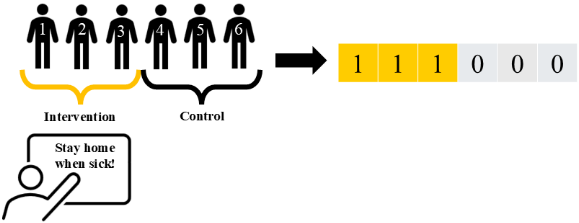

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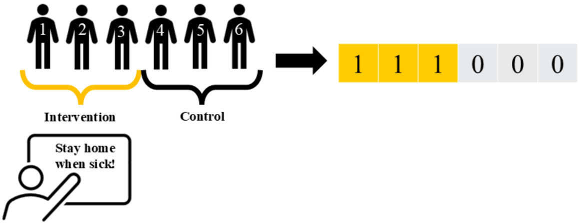

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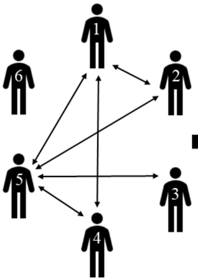

	1	2	3	4	5	6
1	0	1	0	1	1	0
2	1	0	0	0	1	0
3	0	0	0	0	1	0
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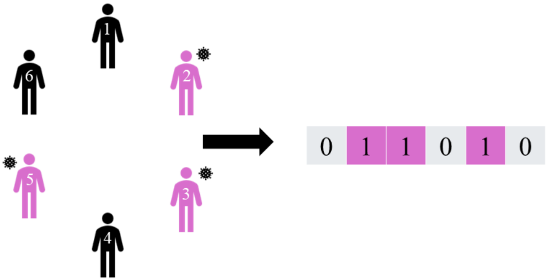

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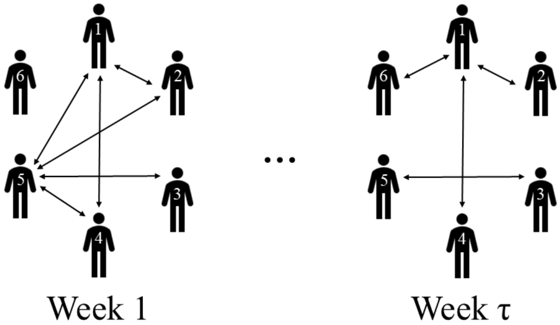
- Network history: $\overline{\mathbf{A}} = \{\mathbf{A}^k\}_{k=1}^{\tau}$



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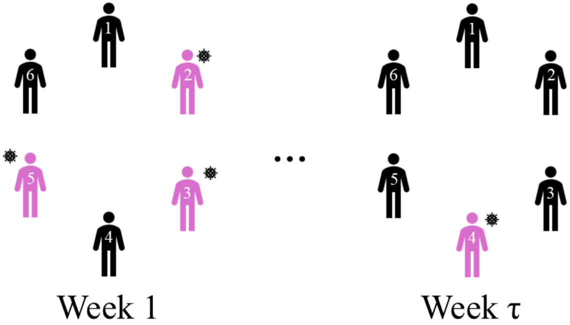

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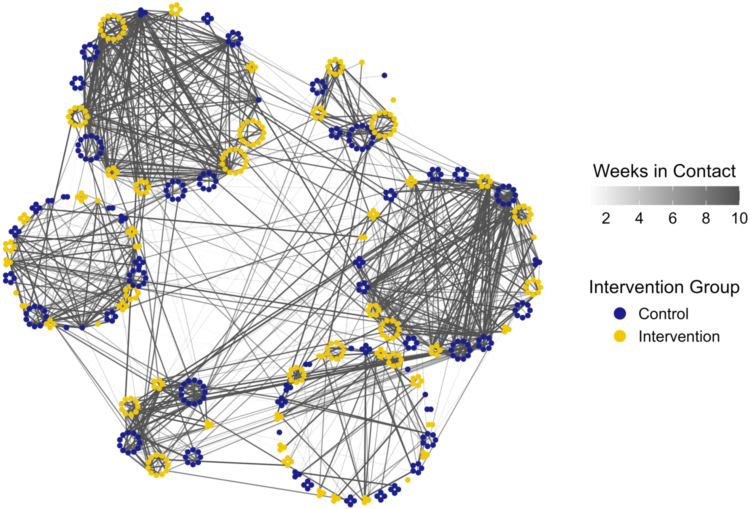

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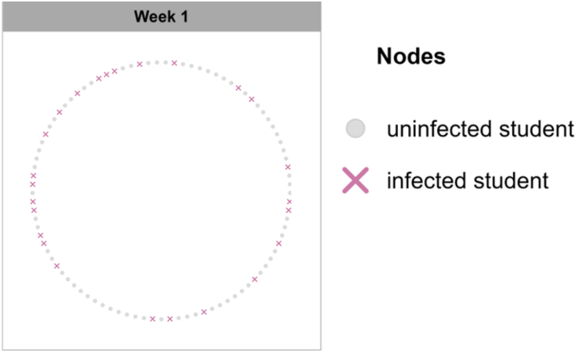

(93 out of 579 students with at least one infection)

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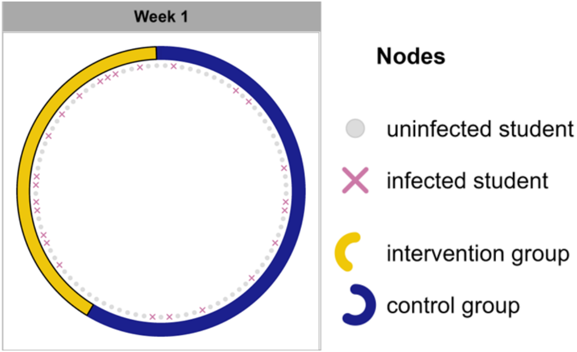

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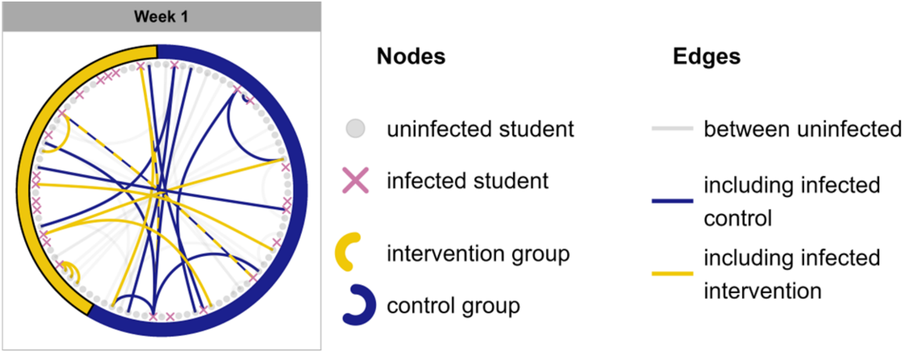

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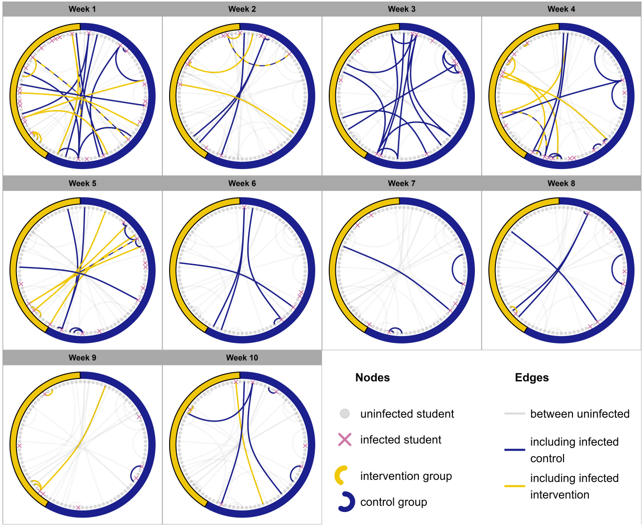

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Potential outcomes are defined for the networks and ILI infections:

$$\overline{\mathbf{A}}(\mathbf{z}), \quad \overline{\mathbf{Y}}(\mathbf{z}) \quad \text{for } \mathbf{z} \in \mathcal{Z}$$

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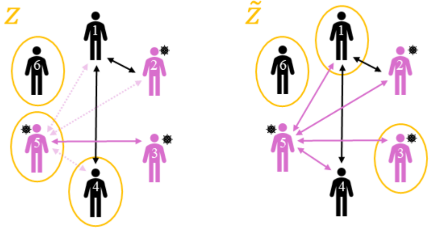
  - potential outcomes for one student depend on intervention assignment of other students (**interference**) (Hudgens and Halloran, 2008)
- Assume **causal consistency**:

$$\overline{\mathbf{A}} = \overline{\mathbf{A}}(\mathbf{Z}) \text{ and } \overline{\mathbf{Y}} = \overline{\mathbf{Y}}(\mathbf{Z})$$

eX-FLU Null Hypotheses

$$H_0^Y : \overline{\mathbf{Y}}(\mathbf{z}) = \overline{\mathbf{Y}}(\widetilde{\mathbf{z}}) \text{ for all } \mathbf{z}, \widetilde{\mathbf{z}} \in \mathcal{Z}$$

("the intervention has no effect on the infections")



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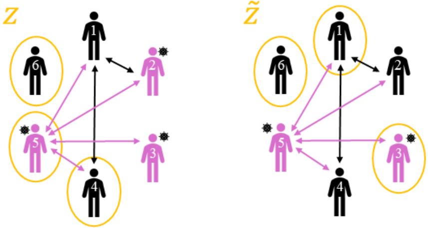

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$$H_0^\sharp = H_0^Y \cap H_0^A : \overline{\mathbf{Y}}(\mathbf{z}) = \overline{\mathbf{Y}}(\widetilde{\mathbf{z}}) \text{ and } \overline{\mathbf{A}}(\mathbf{z}) = \overline{\mathbf{A}}(\widetilde{\mathbf{z}}) \text{ for all } \mathbf{z}, \widetilde{\mathbf{z}} \in \mathcal{Z}$$

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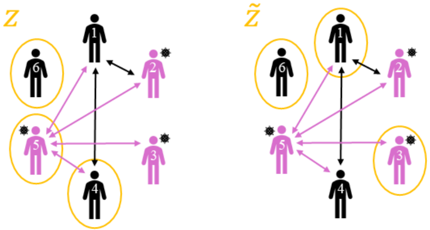

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Previous analyses of the eX-FLU trial (Alexandria et al., 2023) tested $H_0^\sharp$, but no analyses have tested H_0^Y

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Randomization	Networks	Infections	Test Statistic
$\mathbf{z}_1$			
$\mathbf{z}_2$			
$\dots$			
$\mathbf{z}_{\lvert\mathcal{Z}\rvert}$			

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$\mathbf{z}_2$	$\overline{\mathbf{A}}(\mathbf{z}_2) = ?$	$\overline{\mathbf{Y}}(\mathbf{z}_1) = ?$	$T(\mathbf{z}_1, ?, ?)$
$\ldots$	$\ldots$	$\ldots$	$\ldots$
$\mathbf{z}_{\|\mathcal{Z}\|}$	$\overline{\mathbf{A}}(\mathbf{z}_{\|\mathcal{Z}\|}) = ?$	$\overline{\mathbf{Y}}(\mathbf{z}_{\|\mathcal{Z}\|}) = ?$	$T(\mathbf{z}_{\|\mathcal{Z}\|}, ?, ?)$

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- Choose a test statistic $T = T(\mathbf{Z}, \overline{\mathbf{A}}, \overline{\mathbf{Y}})$
- Find the distribution of T under H_0
  - This is relatively easy for a **sharp** null like $H_0^\sharp$

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$\mathbf{z}_1$	$\overline{\mathbf{A}}(\mathbf{z}_1) = ?$	$\overline{\mathbf{Y}}(\mathbf{z}_1) = ?$	$T(\mathbf{z}_1, ?, ?)$
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$\ldots$	$\ldots$	$\ldots$	$\ldots$
$\mathbf{z}_{\lvert\mathcal{Z}\rvert}$	$\overline{\mathbf{A}}(\mathbf{z}_{\lvert\mathcal{Z}\rvert}) = ?$	$\overline{\mathbf{Y}}(\mathbf{z}_{\lvert\mathcal{Z}\rvert}) = ?$	$T(\mathbf{z}_{\lvert\mathcal{Z}\rvert}, ?, ?)$

Testing a Null Hypothesis H_0

"How unlikely are the observed data under H_0?" We can answer this using a **randomization test** (Fisher, 1936; Alexandria et al., 2023; Zhang and Zhao, 2023; Ritzwoller et al., 2024)

- Choose a test statistic $T = T(\mathbf{Z}, \overline{\mathbf{A}}, \overline{\mathbf{Y}})$
- Find the distribution of T under H_0
  - This is relatively easy for a **sharp** null like $H_0^\sharp$
  - This is harder to do for a **nonsharp** null like H_0^Y

Randomization	Networks	Infections	Test Statistic
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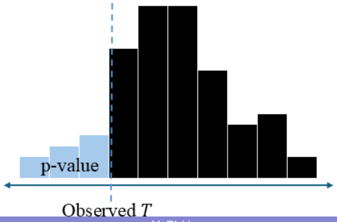
- Compute a p-value: $\rho = F_T(T|H_0)$, where

$$F_T(t|H_0) = \Pr(T \leq t|H_0)$$

  is the **cumulative distribution function** (CDF) of T



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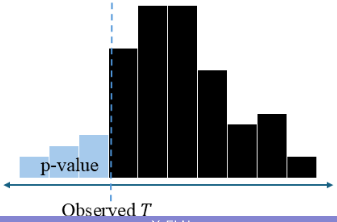
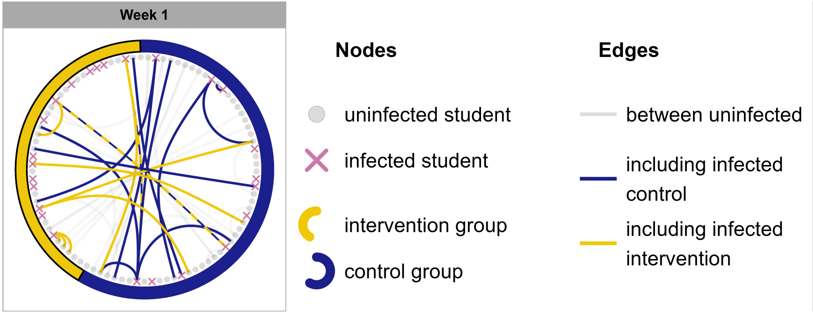


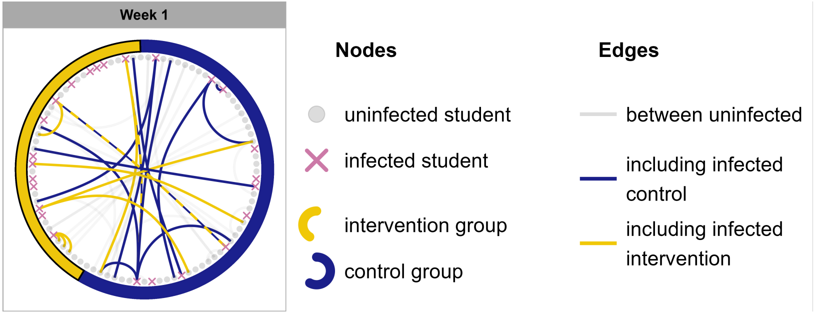
Choice of Test Statistic

- 93 (out of 579) students with at least one infection



Choice of Test Statistic

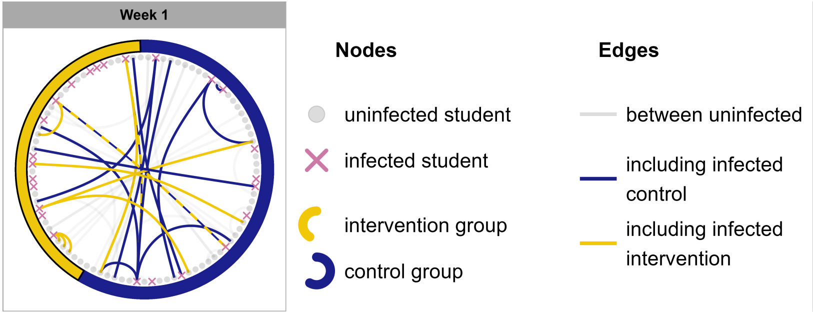
- 93 (out of 579) students with at least one infection
- Bold edges represent possible transmission events



Choice of Test Statistic

- 93 (out of 579) students with at least one infection
- Bold edges represent possible transmission events
  - Proportion of possible transmission events attributable to students in the intervention group:

$$T = \frac{\text{number of yellow edges}}{\text{total number of edges}} = \frac{\sum_{k=2}^{\tau} \sum_{i=1}^{n} \sum_{j \neq i} Z_i Y_i^{k-1} A_{ij}^{k-1}}{\sum_{k=2}^{\tau} \sum_{i=1}^{n} \sum_{j \neq i} Y_i^{k-1} A_{ij}^{k-1}} = 0.359$$



eX-FLU: Hypothesis Test Results

- Test statistic (proportion of possible transmission events attributable to students in the intervention group): $T = 0.359$.

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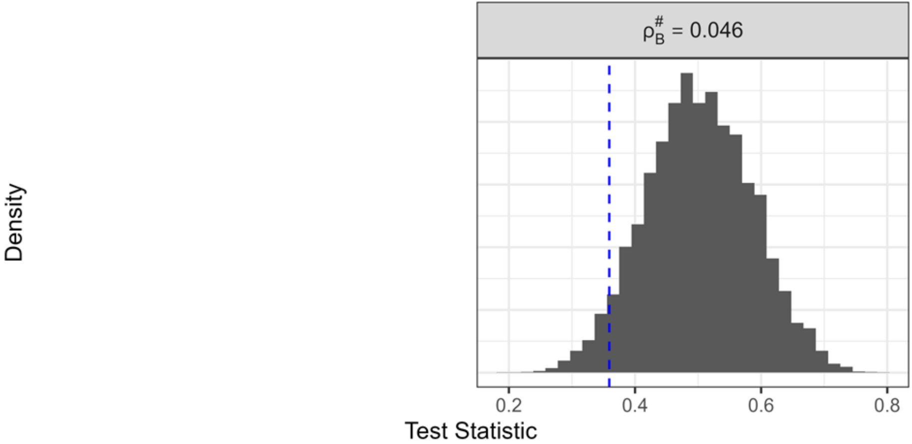

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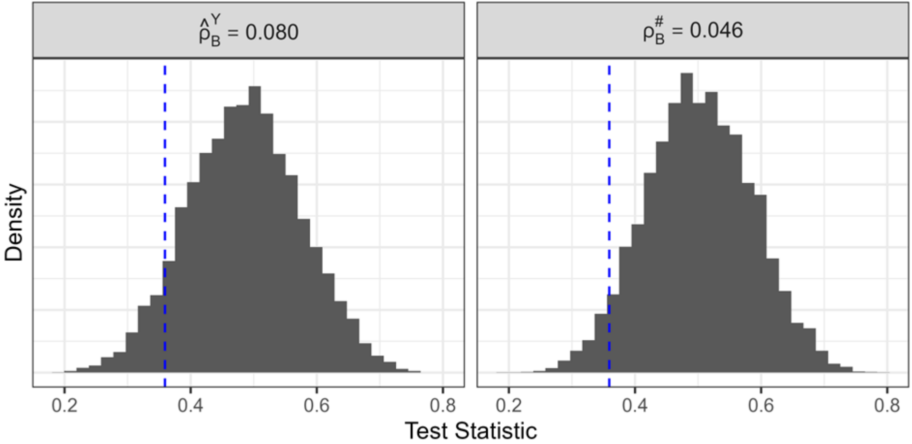

Discussion

Summary:

- Novel testing procedure for testing H_0^Y
- Moderate evidence against H_0^Y in the eX-FLU trial

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- Theoretical results for type I error control
- Empirical results from simulation study

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- Account for measurement error in self-reported networks
- Estimate causal parameter(s)

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Future directions:

- Account for measurement error in self-reported networks
- Estimate causal parameter(s)

Project status:

- Under revision with *PNAS*

Questions?

contact: richabr@wfu.edu

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Supplementary Slides

Testing $H_0^\sharp$

"How unlikely are the observed data under $H_0^\sharp$?"

Testing $H_0^\sharp$

"How unlikely are the observed data under $H_0^\sharp$?" **Randomization test**

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- Choose a test statistic $T = T(\mathbf{Z}, \overline{\mathbf{A}}, \overline{\mathbf{Y}})$

Testing $H_0^\sharp$

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- Choose a test statistic $T = T(\mathbf{Z}, \overline{\mathbf{A}}, \overline{\mathbf{Y}})$
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Randomization	Networks	Infections	Test Statistic
$\mathbf{z}_1$			
$\mathbf{z}_2$			
...			
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Testing $H_0^\sharp$

"How unlikely are the observed data under $H_0^\sharp$?" **Randomization test**

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Randomization	Networks	Infections	Test Statistic
$\mathbf{z}_1$	$\overline{\mathbf{A}}(\mathbf{z}_1) =?$	$\overline{\mathbf{Y}}(\mathbf{z}_1) =?$	$T(\mathbf{z}_1, ?, ?)$
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Testing $H_0^\sharp$

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  - This is possible since $\overline{\mathbf{A}}(\mathbf{z}), \overline{\mathbf{Y}}(\mathbf{z})$ are **imputable** under $H_0^\sharp$

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...	...	...	...
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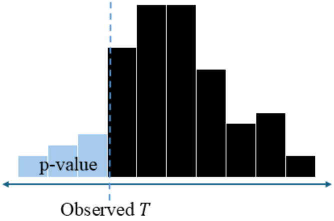
  is the **cumulative distribution function** (CDF) of T



Testing $H_0^\sharp$

"How unlikely are the observed data under $H_0^\sharp$?" **Randomization test**

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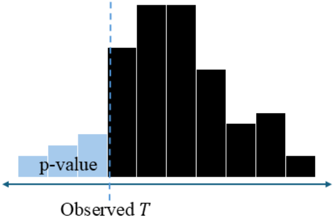
- Reject $H_0^\sharp$ if $\rho^\sharp \leq 0.05$



Testing $H_0^\sharp$

- This test of $H_0^\sharp$ controls the **type I error rate** exactly:

$$\Pr\left(\rho^\sharp \leq \alpha | H_0^\sharp\right) \leq \alpha$$

for any $\alpha \in [0, 1]$, for any sample size, and for any choice of test statistic

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for any $\alpha \in [0, 1]$, for any sample size, and for any choice of test statistic

- The **power** of this test:

$$\Pr\left(\rho^\sharp \le \alpha | H_1^\sharp\right)$$

depends on the choice of test statistic

Testing H_0^Y

"How unlikely are the observed data under H_0^Y?"

Testing H_0^Y

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- Choose a test statistic $T = T(\mathbf{Z}, \overline{\mathbf{A}}, \overline{\mathbf{Y}})$

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- Choose a test statistic $T = T(\mathbf{Z}, \overline{\mathbf{A}}, \overline{\mathbf{Y}})$
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$\mathbf{z}_1$			
$\mathbf{z}_2$			
$\cdots$			
$\mathbf{z}_{\lvert\mathcal{Z}\rvert}$			

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  - This is not possible since $\overline{\mathbf{A}}(\mathbf{z})$ are **not imputable** under H_0^Y

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Testing H_0^Y

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  - This is not possible since $\overline{\mathbf{A}}(\mathbf{z})$ are **not imputable** under $H_0^Y \implies H_0^Y$ is **nonsharp**

Randomization	Networks	Infections	Test Statistic
$\mathbf{z}_1$	$\overline{\mathbf{A}}(\mathbf{z}_1) = ?$	$\overline{\mathbf{Y}}(\mathbf{z}_1) = \overline{\mathbf{Y}}$	$T(\mathbf{z}_1, ?, \overline{\mathbf{Y}})$
$\mathbf{z}_2$	$\overline{\mathbf{A}}(\mathbf{z}_2) = ?$	$\overline{\mathbf{Y}}(\mathbf{z}_2) = \overline{\mathbf{Y}}$	$T(\mathbf{z}_2, ?, \overline{\mathbf{Y}})$
...	...	...	...
$\mathbf{z}_{\lvert\mathcal{Z}\rvert}$	$\overline{\mathbf{A}}(\mathbf{z}_{\lvert\mathcal{Z}\rvert}) = ?$	$\overline{\mathbf{Y}}(\mathbf{z}_{\lvert\mathcal{Z}\rvert}) = \overline{\mathbf{Y}}$	$T(\mathbf{z}_{\lvert\mathcal{Z}\rvert}, ?, \overline{\mathbf{Y}})$

Testing H_0^Y

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- **Proposition**: This procedure will **asymptotically** control the type I error rate

Exponential Family Random Graph Models (ERGMs)

Goal: model a network ($\mathbf{A}$) given covariates ($\mathbf{X}$)

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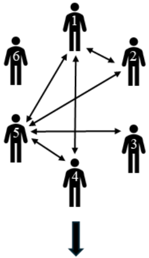

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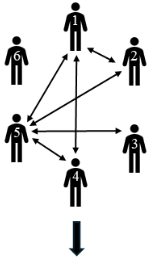

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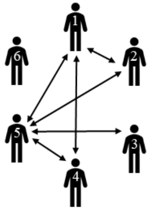

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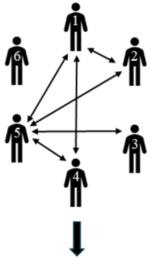

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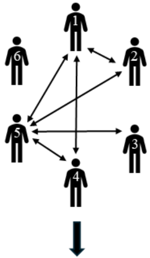

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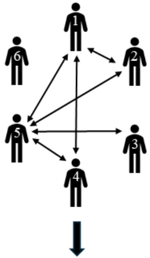

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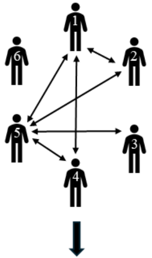

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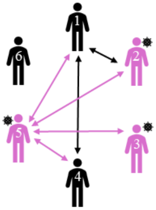

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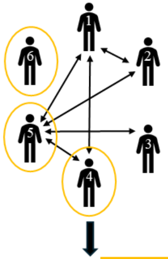

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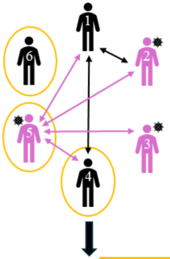

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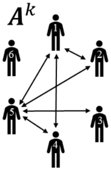
Separable Temporal Exponential Family Random Graph Models

A **STERGM** assumes that, at each time step $k = 1, \ldots, \tau - 1$:

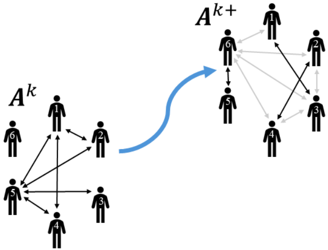


Separable Temporal Exponential Family Random Graph Models

A **STERGM** assumes that, at each time step $k = 1, \ldots, \tau - 1$:

- New edges form according to a **formation** ERGM

$$\Pr_{\theta^+}(\mathbf{A}^{k+} = \mathbf{a}^{k+} | \mathbf{A}^k = \mathbf{a}^k, \mathbf{X}^{k+1} = \mathbf{x}^{k+1}) = \frac{\exp\left(\theta^+ \mathbf{g}^+(\mathbf{a}^{k+}, \mathbf{a}^k, \mathbf{x}^{k+1})\right)}{\kappa_{\mathbf{g}^+}(\theta^+, \mathcal{A}^+(\mathbf{a}^k), \mathbf{a}^k, \mathbf{x}^{k+1})},$$

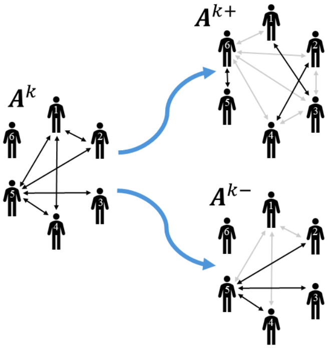


Separable Temporal Exponential Family Random Graph Models

A **STERGM** assumes that, at each time step $k = 1, \ldots, \tau - 1$:

- New edges form according to a **formation** ERGM
- Old edges persist according to a **persistence** ERGM

$$\Pr_{\theta^-}(\mathbf{A}^{k-} = \mathbf{a}^{k-} | \mathbf{A}^k = \mathbf{a}^k, \mathbf{X}^{k+1} = \mathbf{x}^{k+1}) = \frac{\exp\left(\theta^- \mathbf{g}^-(\mathbf{a}^{k-}, \mathbf{a}^k, \mathbf{x}^{k+1})\right)}{\kappa_{\mathbf{g}^-}(\theta^-, \mathcal{A}^-(\mathbf{a}^k), \mathbf{a}^k, \mathbf{x}^{k+1})},$$



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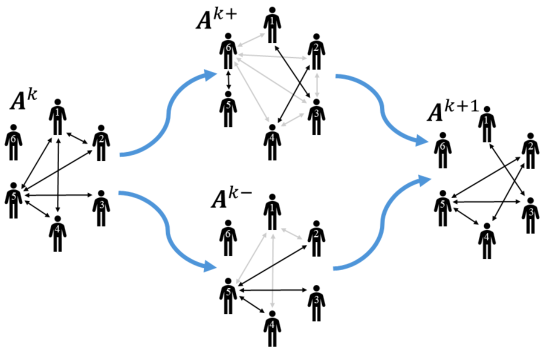
$$\mathbf{A}^{k+1} = \underbrace{\mathbf{A}^{k}}_{\text{previous network}} \cup \underbrace{(\mathbf{A}^{k+} - \mathbf{A}^{k})}_{\text{new edges formed}} - \underbrace{(\mathbf{A}^{k} - \mathbf{A}^{k-})}_{\text{old edges not persisting}}$$



Separability of STERGMs

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$$
\begin{tikzcd}
 & \mathbf{A}^{1+} \arrow[dr] & & \mathbf{A}^{2+} \arrow[dr] & & \mathbf{A}^{3+} \arrow[dr] & \\
\mathbf{A}^{1} \arrow[ur] \arrow[rr] \arrow[dr] & & \mathbf{A}^{2} \arrow[ur] \arrow[rr] \arrow[dr] & & \mathbf{A}^{3} \arrow[ur] \arrow[rr] \arrow[dr] & & \mathbf{A}^{4} \\
 & \mathbf{A}^{1-} \arrow[ur] & & \mathbf{A}^{2-} \arrow[ur] & & \mathbf{A}^{3-} \arrow[ur] &
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2. the parameter space for $\theta = (\theta^{+}, \theta^{-})$ is the product of the parameter spaces for θ^{+} and θ^{-}

$$
\begin{tikzcd}
& \mathbf{A}^{1+} \arrow[dr] & & \mathbf{A}^{2+} \arrow[dr] & & \mathbf{A}^{3+} \arrow[dr] & \\
\mathbf{A}^{1} \arrow[ur] \arrow[rr] \arrow[dr] & & \mathbf{A}^{2} \arrow[ur] \arrow[rr] \arrow[dr] & & \mathbf{A}^{3} \arrow[ur] \arrow[rr] \arrow[dr] & & \mathbf{A}^{4} \\
& \mathbf{A}^{1-} \arrow[ur] & & \mathbf{A}^{2-} \arrow[ur] & & \mathbf{A}^{3-} \arrow[ur] &
\end{tikzcd}
$$

Testing H_0^Y

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3. compute the plug-in p-value testing H_0^Y:

$$\widehat{p}_B^Y = B^{-1} \sum_{b=1}^{B} \mathbb{1}(T_b \leq T)$$

Type I Error Control

- **Proposition 2.1**: Let T_N be a test statistic with CDF F_N under H_0.
  1. Under H_0, $F_N(T_N)$ stochastically dominates a Uniform$(0, 1)$ distribution for any N.
  2. If the test statistic T_N has a continuous limiting distribution, then $F_N(T_N) \to^d$ Uniform$(0, 1)$.
- **Corollary 2.2**:
  1. Under $H_0^\sharp$, the sharp null p-value $\rho_N^\sharp$ stochastically dominates a Uniform$(0, 1)$ distribution for any N.
  2. Under H_0^Y, the oracle p-value ρ_N^Y stochastically dominates a Uniform$(0, 1)$ distribution for any N.
  3. If the test statistic T_N has a continuous limiting distribution, then $\rho_N^\sharp \to^d$ Uniform$(0, 1)$ under $H_0^\sharp$ and $\rho_N^Y \to^d$ Uniform$(0, 1)$ under H_0^Y.

Type I Error Control

- **Proposition 2.3**: Let $q(\mathbf{a}, \mathbf{z}, \theta) \equiv \Pr\{\mathbf{A}(\mathbf{z}) = \mathbf{a}; \theta\}$ denote the PMF of the distribution of stochastic potential networks $\mathbf{A}(\mathbf{z})$ at parameter value θ, let $\widehat{\theta}_N$ denote the estimator of θ, and let $F_N(\cdot; \theta)$ denote the CDF of the test statistic T_N at θ, with limiting CDF $F(\cdot; \theta)$. Let θ_0 denote the true value of θ. Assume the following:

  (A1) $\widehat{\theta}_N \to^p \theta_0$

  (A2) $F(t; \theta_0)$ is continuous in t on $\mathbb{R}$

  (A3) there exists a $\delta_0 > 0$ such that

  $$\sup_{\theta \in B_{\delta_0}(\theta_0)} \sup_{t \in \mathbb{R}} |F_N(t; \theta) - F(t; \theta)| \to 0$$

  (A4) $F(t; \theta)$ is continuous in θ at θ_0 uniformly in t, i.e.,

  $$\lim_{\theta \to \theta_0} \sup_{t \in \mathbb{R}} |F(t; \theta) - F(t; \theta_0)| = 0$$

  Then the plug-in p-value $\widehat{\rho}_N^Y$ converges in distribution to Uniform$(0, 1)$.

Type I Error Control

- **Proposition 2.4**: Let $\rho_N = F_N(T_N)$ for test statistic T_N and CDF F_N (not necessarily the true CDF of T_N). Let $T_N^* = h_N(T_N)$ for a sequence of deterministic, strictly increasing functions h_N. Define $F_N^*(t) = F_N\{h_N^{-1}(t)\}$, the (not necessarily true) CDF of the transformed test statistic, and let $\rho_N^* = F_N^*(T_N^*)$. Then $\rho_N^* = \rho_N$.
- **Corollary 2.5**: Let h_N be a sequence of deterministic, strictly increasing functions.
    1. If the hypotheses of Proposition 2.1 are met for a test statistic T_N, then the results also hold for $T_N^* = h_N(T_N)$.
    2. If the hypotheses of Proposition 2.3 are met for $T_N, F_N(\cdot;\theta)$, then the results also hold for $T_N^* = h_N(T_N), F_N^*(\cdot;\theta) = F_N\{h_N^{-1}(\cdot);\theta\}$.

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Goal: use simulated data to investigate the performance of the testing procedure

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  1. $H_0^\sharp$: no effect of intervention on networks or infection
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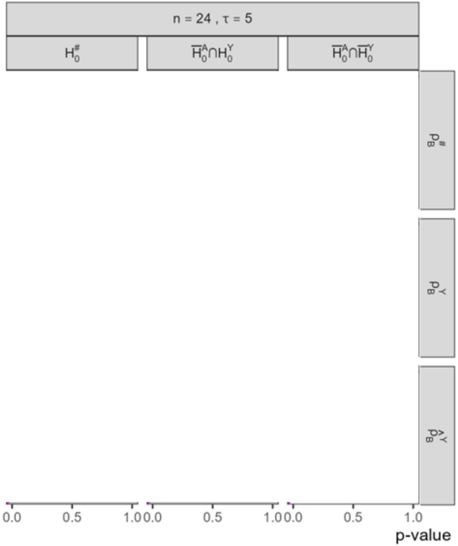
- Three testing procedures:
  1. $\rho_B^\sharp$: testing $H_0^\sharp$
  2. ρ_B^Y: testing H_0^Y using known q
  3. $\widehat{\rho}_B^Y$: testing H_0^Y using estimated q

Simulation Results: Empirical CDF of P-Values



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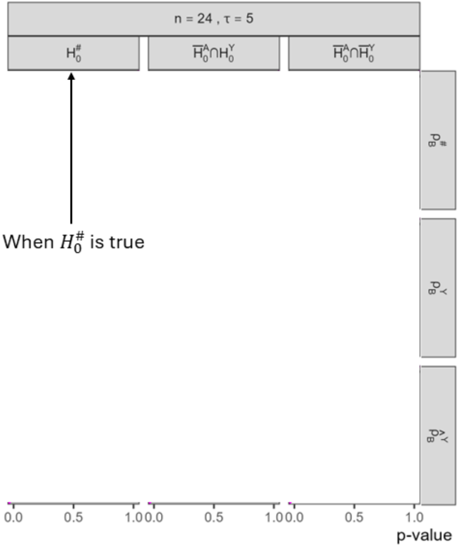

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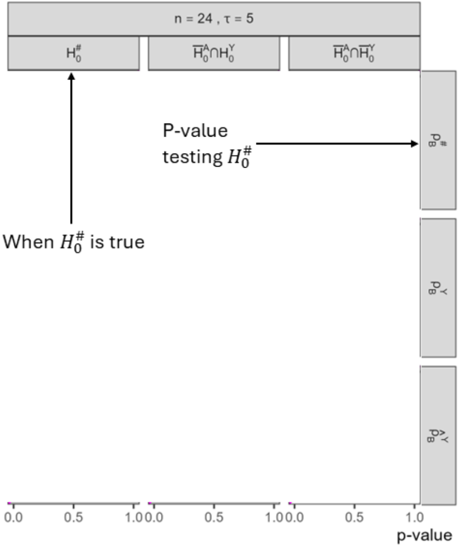

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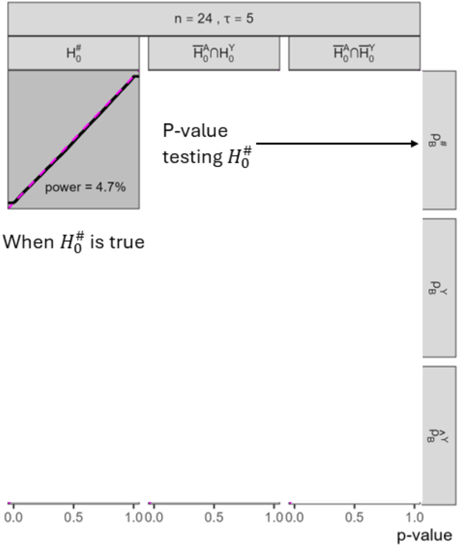

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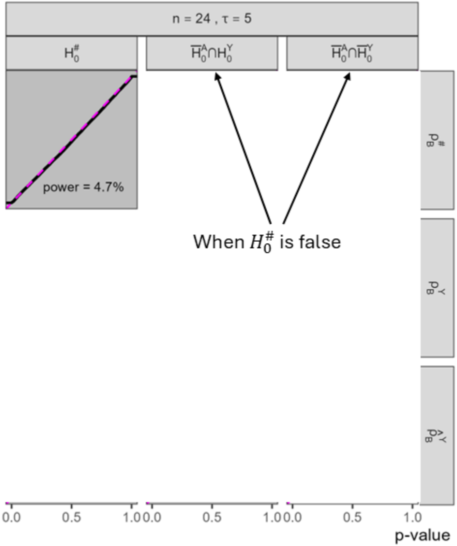

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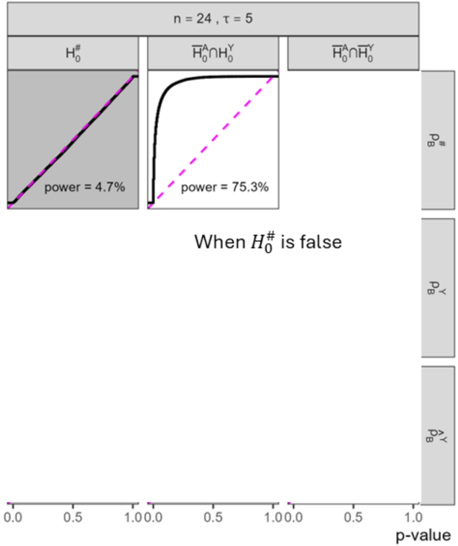

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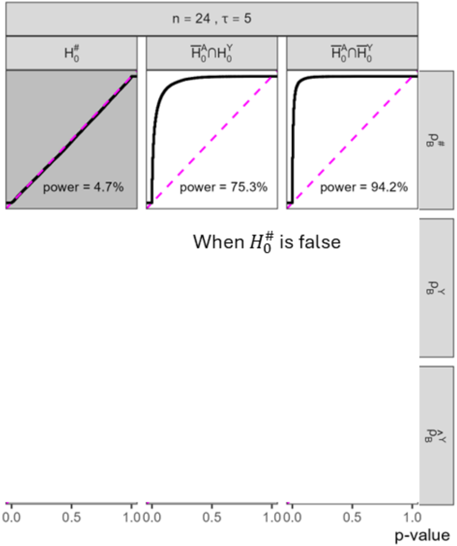

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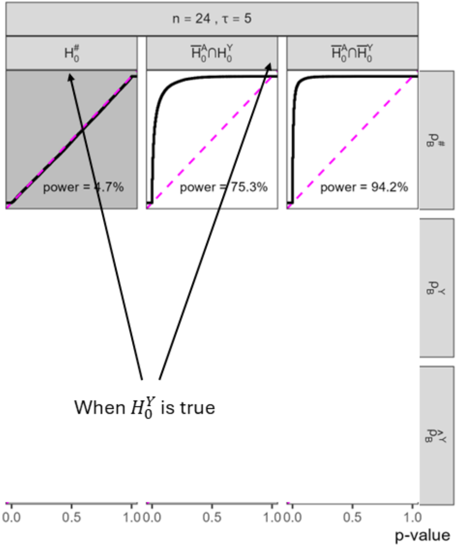

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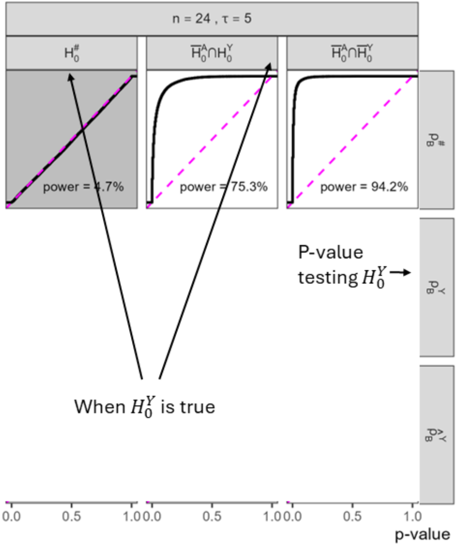

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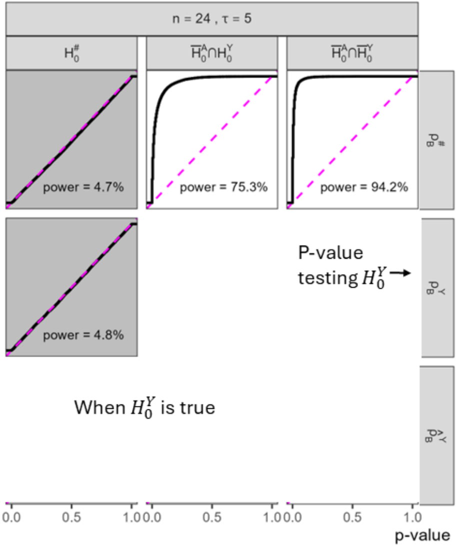

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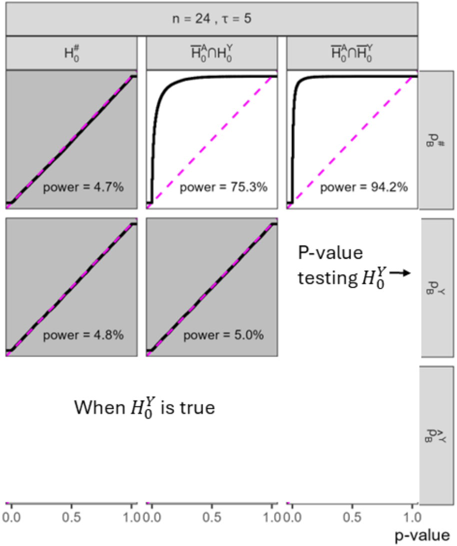

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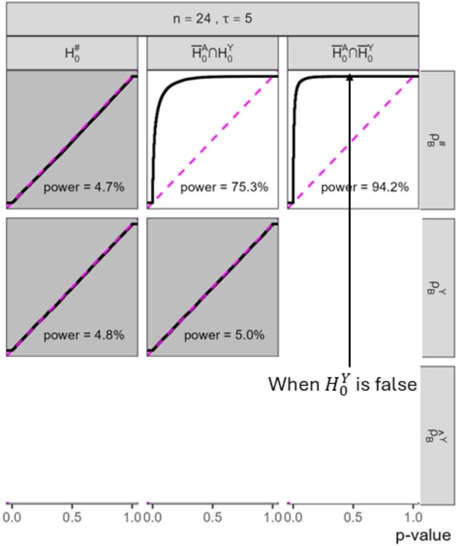

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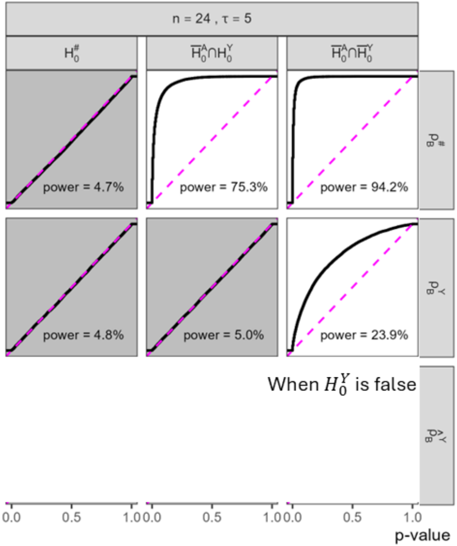

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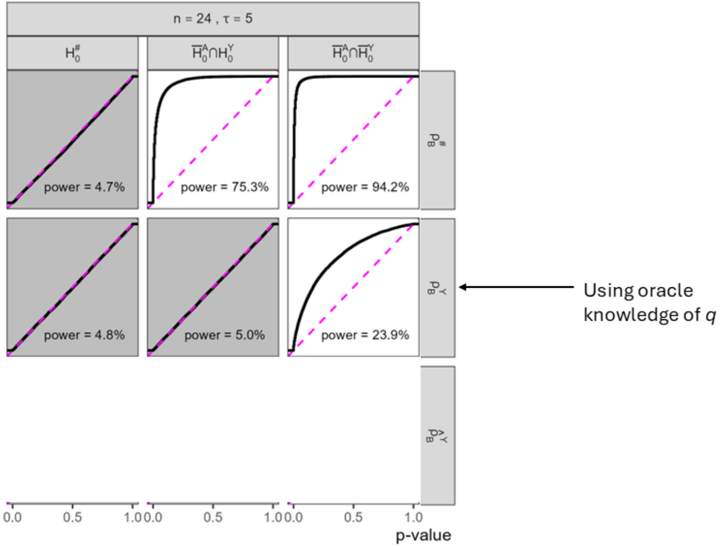

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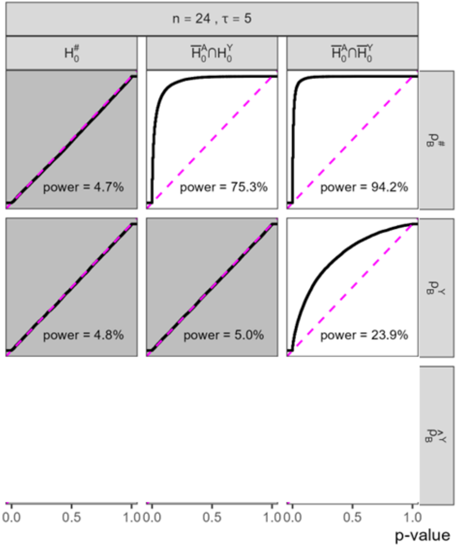

Using oracle knowledge of q

Using estimated q

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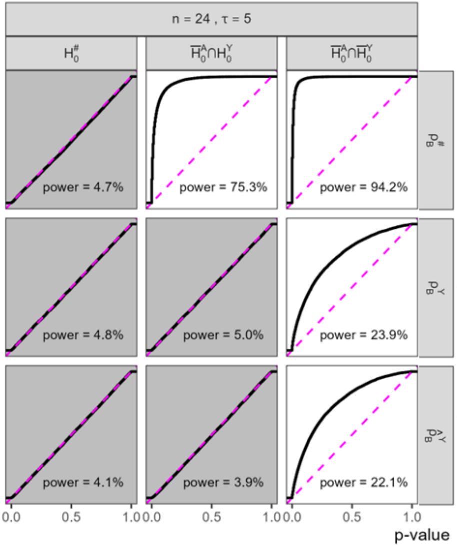

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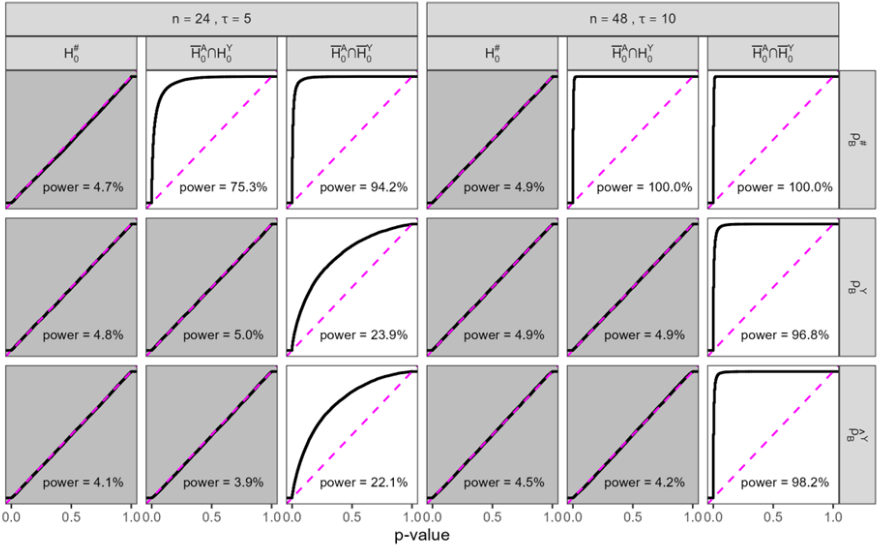


eX-FLU: STERGM Specification

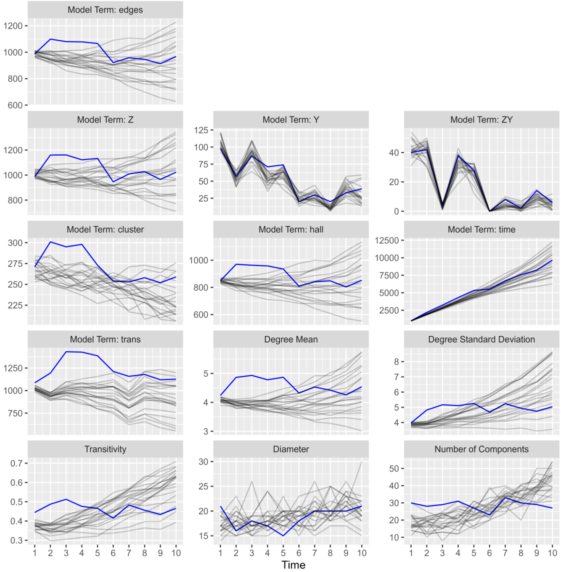
$\mathbf{g}^{+}(\mathbf{a}^{k+}, \mathbf{a}^{k}, \mathbf{x}^{k+1}) =$

$$
\begin{bmatrix}
g^{+}_{\text{edges}}(\mathbf{a}^{k+}, \mathbf{x}^{k+1}) \\
g^{+}_{Z}(\mathbf{a}^{k+}, \mathbf{x}^{k+1}) \\
g^{+}_{Y}(\mathbf{a}^{k+}, \mathbf{x}^{k+1}) \\
g^{+}_{ZY}(\mathbf{a}^{k+}, \mathbf{x}^{k+1}) \\
g^{+}_{\text{room}}(\mathbf{a}^{k+}, \mathbf{x}^{k+1}) \\
g^{+}_{\text{cluster}}(\mathbf{a}^{k+}, \mathbf{x}^{k+1}) \\
g^{+}_{\text{hall}}(\mathbf{a}^{k+}, \mathbf{x}^{k+1}) \\
g^{+}_{\text{time}}(\mathbf{a}^{k+}, \mathbf{x}^{k+1}) \\
g^{+}_{\text{trans}}(\mathbf{a}^{k}, \mathbf{x}^{k+1})
\end{bmatrix}
=
\begin{bmatrix}
\sum_{i,j} a_{ij}^{k+} \\
\sum_{i,j} a_{ij}^{k+}(z_i + z_j) \\
\sum_{i,j} a_{ij}^{k+}(y_i^{k+1} + y_j^{k+1}) \\
\sum_{i,j} a_{ij}^{k+}(z_i y_i^{k+1} + z_j y_j^{k+1}) \sum_{i,j} a_{ij}^{k+} \mathbb{1}(i,j \text{ roommates}) \\
\sum_{i,j} a_{ij}^{k+} \mathbb{1}(i,j \text{ cluster-mates}) \\
\sum_{i,j} a_{ij}^{k+} \mathbb{1}(i,j \text{ hall-mates}) \\
\sum_{i,j} a_{ij}^{k+} \\
\sum_{i,j} \log\left(1 + \sum_{l} a_{il}^{k} a_{jl}^{k}\right)
\end{bmatrix}
$$

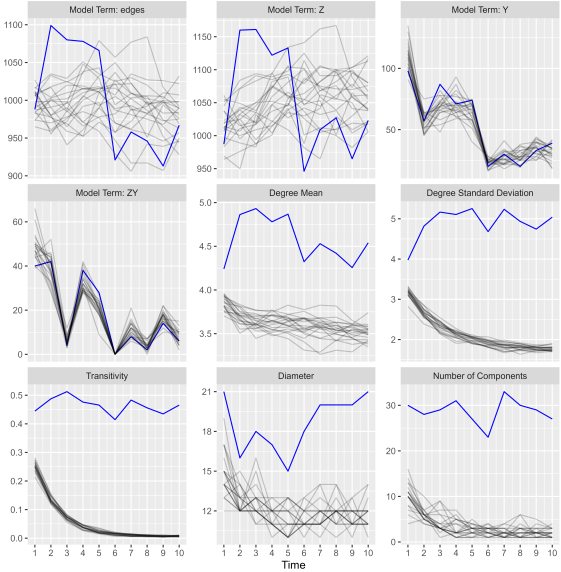
eX-FLU: STERGM Results

Model	Parameter	Estimate (95% Confidence Interval)
Formation	$\theta^{+}_{\text{edges}}$	-7.85 (-8.00, -7.71)
	θ^{+}_{Z}	0.09 (0.02, 0.15)
	θ^{+}_{Y}	0.71 (0.50, 0.93)
	θ^{+}_{ZY}	-0.42 (-0.79, -0.05)
	θ^{+}_{room}	∞ (offset)
	$\theta^{+}_{\text{cluster}}$	2.02 (1.90, 2.14)
	θ^{+}_{hall}	1.71 (1.57, 1.84)
	θ^{+}_{time}	-0.09 (-0.11, -0.07)
	$\theta^{+}_{\text{trans}}$	2.88 (2.82, 2.94)
Persistence	$\theta^{-}_{\text{edges}}$	0.16 (-0.02, 0.33)
	θ^{-}_{Z}	0.03 (-0.03, 0.10)
	θ^{-}_{Y}	-0.06 (-0.32, 0.21)
	θ^{-}_{ZY}	-0.11 (-0.55, 0.33)
	θ^{-}_{room}	∞ (offset)
	$\theta^{-}_{\text{cluster}}$	0.26 (0.13, 0.39)
	θ^{-}_{hall}	0.15 (0.00, 0.31)
	θ^{-}_{time}	0.04 (0.02, 0.06)
	$\theta^{-}_{\text{trans}}$	0.42 (0.35, 0.48)

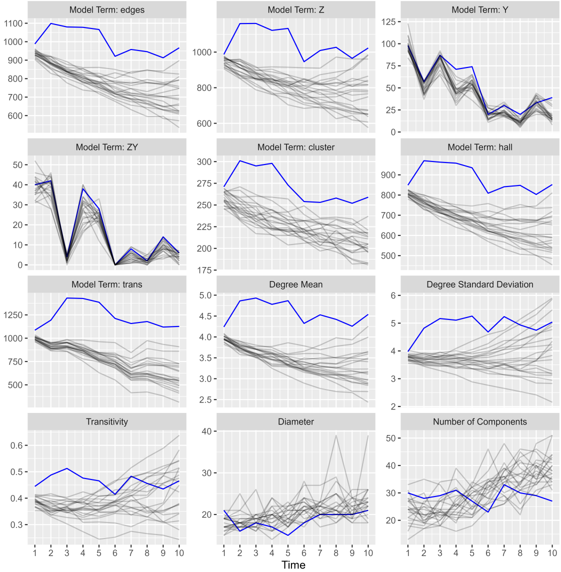
eX-FLU Diagnostics: STERGM



eX-FLU Diagnostics: Alternate STERGM 1



eX-FLU Diagnostics: Alternate STERGM 2



eX-FLU Sensitivity Analysis